Hence $B'K'=FK'$. $\therefore$ The locus of K' is a parabola.

COROLLARY 1. *The diagonal of the rhombus is a tangent to the curve in every position; and the second diagonal is parallel to the normal.*

COROLLARY 2. *When the diagonal is parallel to the radius it is an asymptote.*

COROLLARY 3. *If a point in one side of a movably pivoted rhombus, at a given distance from the vertex, is constrained to move in the circumference of a directing circle, and a point in the adjacent side equidistant from the same vertex is fixed in the diameter (or diameter produced), the locus of the intersection of the diagonal (produced if necessary) through the other two vertices with the radius (or radius produced) of the directing circle is a conic.*

COROLLARY 4. *A conic is the locus of a point which moves so that the ratio of its distances from a fixed circle and a fixed point in its diameter (or diameter produced) is equal to unity.*

The linkage is very easily constructed by using thin strips of wood or metal for each line in the above figure except the diameter of the directing circle, which should be strong enough to support the rest of the linkage without bending. The links representing the diagonal and the radius should be slotted, then there will be an opening at their intersection in which a pencil can be inserted to describe the curve.

The points F and F' represent the foci, one of which, the point F, should be made to slide along the diameter. Then a change in the curve due to a change in the relative position of the foci is made evident.

If the diagonal DI passes through the point F', and an extra link DE is attached to the vertex D with the point E in the diameter, but so situated that $DE=EF'$, then the vertex I will describe a straight line.

For we have $F'D\times F'I=$constant. Consequently, if the vertex D describes a circle, the point I must describe its inverse.

The extreme simplicity of this linkage reduces the geometry of the conics to that of the rhombus and the circle.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

Problem 185 was also solved by A. H. Holmes, Brunswick, Maine.

187. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Express by radicals the roots of $x^7+px^5+\frac{2}{7}p^2x^3+\frac{1}{49}p^3x+r=0$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $x=y+z$. Then the equation becomes

$$y^7+z^7+(7yz+p)[y^5+z^5+(3yz+\tfrac{2}{7}p)(y^3+z^3)$$
$$+(5y^2z^2+\tfrac{5}{7}pyz+\tfrac{1}{49}p^2)(y+z)]+r=0.$$

Now x may be decomposed into two parts, y and z, in an infinite number of ways; we may, therefore, suppose y and z are such as to satisfy the condition $7yz+p=0$.

$\therefore\ yz=-\frac{1}{7}p,\ y^7+z^7=-r.$

Let $y^7=a$, $z^7=b$. $\therefore\ a+b=-r,\ ab=-(\frac{1}{7}p)^7$.

$\therefore$ a and b are the roots of the equation $u^2+ru-(\frac{1}{7}p)^7=0$.

$\therefore\ a=-\frac{1}{2}r+\surd[\frac{1}{4}r^2+(\frac{1}{7}p)^7],\ b=-\frac{1}{2}r-\surd[\frac{1}{4}r^2+(\frac{1}{7}p)^7]$.

Let ω be an imaginary seventh root of unity, so that

$$\omega=\tfrac{1}{2}[A\pm\surd(A^2-4)],\ A=\tfrac{1}{3}\sqrt[3]{\tfrac{7}{2}}[\sqrt[3]{(1+3\surd-3)}+\sqrt[3]{(1-3\surd-3)}-\sqrt[3]{\tfrac{2}{7}}].$$

Then the required seven roots of the equation are

$$\sqrt[7]{a}+\sqrt[7]{b},\quad \omega\sqrt[7]{a}+\omega^6\sqrt[7]{b},\quad \omega^2\sqrt[7]{a}+\omega^5\sqrt[7]{b},\quad \omega^3\sqrt[7]{a}+\omega^4\sqrt[7]{b},$$
$$\omega^6\sqrt[7]{a}+\omega\sqrt[7]{b},\quad \omega^5\sqrt[7]{a}+\omega^2\sqrt[7]{b},\quad \omega^4\sqrt[7]{\iota}+\omega^3\sqrt[7]{b}.$$

188. Proposed by GUY SCHUYLER.

$xy+ab=2ax,\quad x^2y^2+a^2b^2=2b^2y^2.$

Solution by O. W. ANTHONY, Head of Mathematical Department, DeWitt Clinton High School, New York City.

By squaring the first equation and subtracting from the second we get $y=xa/b$ and $y=-2xa/b$. Whence easily $x=b,\ \frac{1}{2}(-1\pm\surd3)b$. $y=a(1\mp\surd3)b/a$.

Also solved by G. W. Greenwood, B. A., Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.; G. W. Drake, Fayetteville, Ark.; Charles E. Barrett, Louisville, Ky.; H. F. MacNeish, A. B., Instructor in Mathematics in the University High School, Chicago, Ill.; L. E. Newcomb, Los Gatos, Cal.; G. B. M. Zerr, A. M., Ph. D., Parsons, W. Va., and J. Scheffer, Kee Mar College, Hagerstown, Md.

GEOMETRY.

Problem 203 was also solved by Henry A. Converse, Ph. D., Instructor in Mathematics, Johns Hopkins University, Baltimore, Md.

208. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

Tangents drawn to two confocal parabolae from the point on the common tangent intersect at the same angle as the axes of the parabolae.

I. Solution by G. W. GREENWOOD, A. M. (Oxon), Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

PQ is the common tangent; S, the common focus. Draw PA, PA' parallel to the axes; PT, PT' are the other tangents from P; T, T' being the points of tangency. Join PS.